6 Multi-level PAM and its MAP Detector

Thursday, July 18, 2013 1:41 PM

Multi-level PAM and its MAP Detector

$$\hat{s}_{MAP}(r) = \arg\max_{s} \; p_s f_N(r-s)$$

Example: There are four possible values for S

↳ $-6, -3, 3, 6$ (s_1, s_2, s_3, s_4)

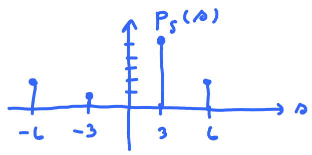
$p_i = P[S = s_i]$

$p_1 = 0.2$

$p_2 = 0.1$

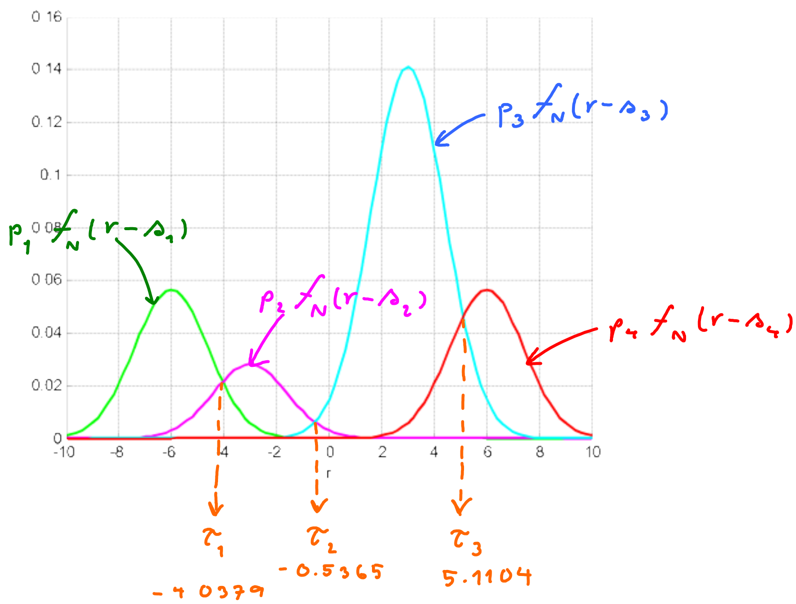
$p_3 = 0.5$

$p_4 = 0.2$



Assume

$N \sim \mathcal{N}(0, \sigma^2)$



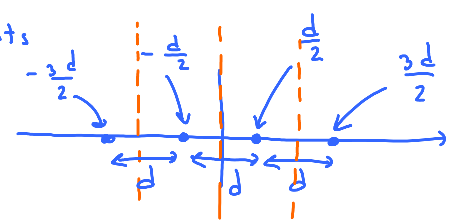
There is no systematic pattern for the τ values.
Therefore, finding $P(\mathcal{E})$ will be quite tedious.
See HW2 for an example of $P(\mathcal{E})$ evaluation.

"standard" Multi-level PAM

→ ① The distances btw adjacent points are all "d".
(and center around 0)

M = # points

$M = 4$



→ ② Assume equiprobable s's.

Average energy per symbol $= \frac{1}{4} \times \left(-\frac{3d}{2}\right)^2 + \frac{1}{4}\left(-\frac{d}{2}\right)^2 + \frac{1}{4}\left(\frac{d}{2}\right)^2 + \frac{1}{4}\left(\frac{3d}{2}\right)^2$

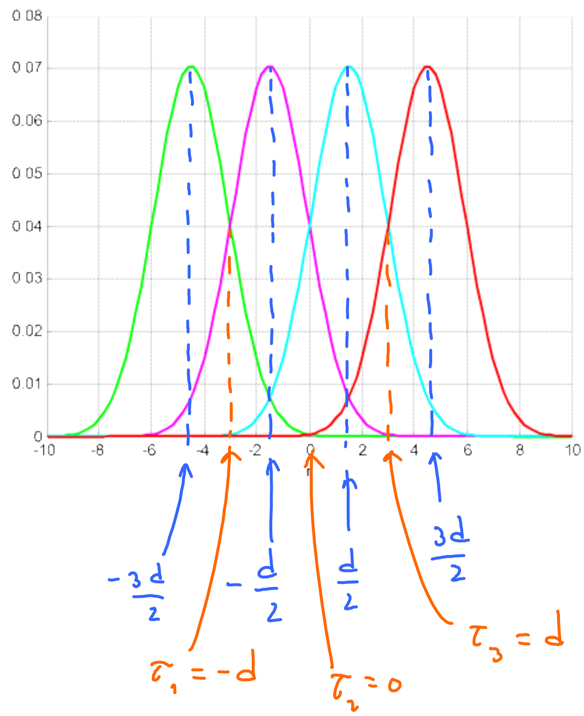
$= \frac{5}{4} d^2 \equiv E_s$

In this case, one symbol $= \log_2 4 = 2$ bits

Average energy per bit $\equiv E_b \equiv \frac{E_s}{\log_2 M} = \frac{(5/4)d^2}{2} = \frac{5}{8}d^2. \Rightarrow d = 2\sqrt{\frac{2}{5}E_b}$

Assume: $N \sim \mathcal{N}(0, \sigma^2)$ ← σ_N^2

MAP Detector: $\hat{s}_{MAP}(r) = \arg\max_{s} \; p_s f_N(r-s)$ ← $s = s_1, s_2, s_3, s_4$



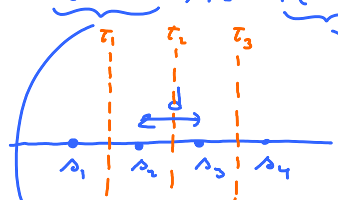
$$\hat{s}_{MAP}(r) = \begin{cases} s_1, & r < -d \\ s_2, & -d \le r < 0 \\ s_3, & 0 \le r < d \\ s_4, & r \ge d \end{cases}$$

$$P(\mathcal{E}) = \underbrace{P(\mathcal{E} \mid s = s_1)}_{P[N > \tau_1 - s_1]} \overbrace{p_1}^{1/4} + \underbrace{P(\mathcal{E} \mid s = s_2)}_{2 \times Q\left(\frac{d/2}{\sigma}\right)} \overbrace{p_2}^{1/4} + \underbrace{P(\mathcal{E} \mid s = s_3)}_{2 \times Q\left(\frac{d/2}{\sigma}\right)} \overbrace{p_3}^{1/4} + \underbrace{P(\mathcal{E} \mid s = s_4)}_{P[N < \tau_3 - s_4]} \overbrace{p_4}^{1/4}$$

$P[N > \tau_1 - s_1]$

$= P[N > \frac{d}{2}]$

$= Q\left(\frac{d/2}{\sigma}\right)$



$P[N < \tau_3 - s_4]$

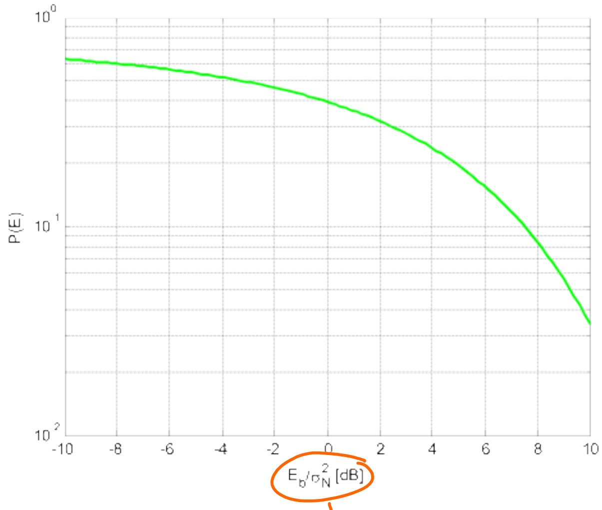
$= P[N < -\frac{d}{2}]$

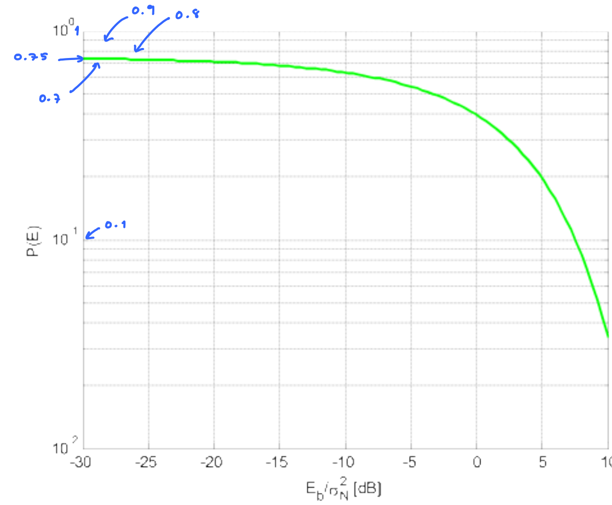
$= Q\left(\frac{d/2}{\sigma}\right)$

$= \frac{3}{2} Q\left(\frac{d}{2\sigma}\right)$

$= \frac{3}{2} Q\left(\frac{1}{\cancel{2}\sigma} \cancel{2}\sqrt{\frac{2}{5}E_b}\right) = \frac{3}{2} Q\left(\sqrt{\frac{2}{5}\frac{E_b}{\sigma^2}}\right)$ ← signal-to-noise ratio

(SNR)

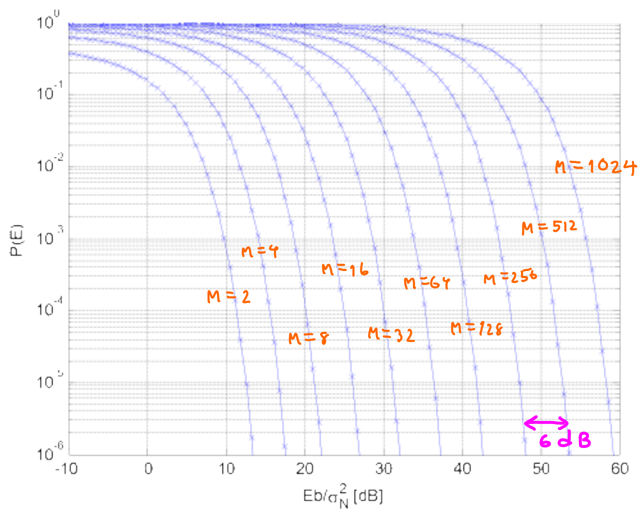




$$\frac{E_b}{\sigma^2}[dB] = 10\log_{10}\frac{E_b}{\sigma^2}$$

$$\frac{E_b}{\sigma^2} = 10^{\frac{dB}{10}}$$

For general M,



So, one bit increase in the amount of bits conveyed per symbol requires 6 dB increase in SNR.

In HW 2, we look at the same system but the noise is assumed to be exponential.